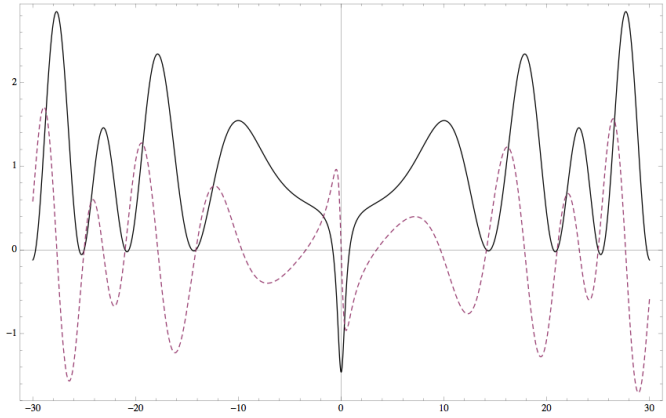

1. A trip to Göttingen

Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. 'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean.

Godfrey H. Hardy (1877-1947), British mathematician.

1. During the flight that brings me to Frankfurt from Barcelona I reread the biography of Carl Friedrich Gauss(1), written by Waldo Dunnington in 1955(2). It is one of the best biographies of this great mathematician and astronomer, who decisively contributed to the progress of mathematics during the late eighteenth and first half of the nineteenth century. I close the book for a few minutes and think about my trip. I organize my ideas. My friend Joan Marcos, a computer engineer whom I have known for many years, and whose curiosity has made him want to get involved in this attractive world of prime numbers, accompanies me.

His analytical capacity to solve problems, crumble them and achieve brilliant solutions has always amazed me. The affinity that we share has made us do this trip together.

And, after several weeks of 2011 preparing for this, we head to Göttingen, a German city located in the center of the country, whose university shone as we have rarely seen a university shine in the firmament of mathematics. Two brilliant mathematicians, among others, lived in this city and developed their work: the aforementioned Gauss and Bernhard Riemann. These two men have written more than half of what is known today on prime numbers. As it has sometimes happened in history, his work was not completed. Gauss had a very long life for his time, for he lived between 1777 and 1855. However, Riemann lived only 39 years, between 1826 and 1866, but his life was so fertile it stunned the world, as he established the mathematical foundations of great progress that took place much later, in the twentieth century.

Market Square (Marktplatz) *Göttingen*.

Photo by the author.

We agreed to a series of meetings with the professor of number theory Samuel J. Patterson, at the University of Göttingen, and with Dr. Axel Wittmann, Director of the Gauss-Gesellschaft (Gauss Association). We shared preliminary discussions on the life and work of Gauss and Riemann, during which I was inoculated the poison of curiosity. Another reason for our trip is a visit to the *Handschriftenabteilung* (Manuscript Department) of the University of Göttingen, where the most important original documents, manuscripts, portraits and books of the time are kept.

And that is the first key element: the time period. We are going to focus on a part of mathematical history, spanning from 1707, with the birth of the Swiss mathematician Leonhard Euler, to this day. We, therefore, have before us more than three centuries, during which mathematics, as we know them today, were developed. Without wishing to belittle, of course, earlier mathematicians such as Isaac Newton, René Descartes, Gottfried Leibniz and many others, I am focusing on those who most successfully undertook the subject of this book: the knowledge of prime numbers. And among them, he who decisively took a giant step in the study thereof due to the conclusions he reached: Georg Friedrich Bernhard Riemann, our protagonist.

2. Göttingen is located in central Germany, and the best way to get there is by train. So, on arrival in Frankfurt, we took the *Inter-City Express,* that left us in the city center in a couple of hours, barely a few meters away from the medieval buildings and from the ancient city wall, now converted into a walk. Among them is the University, the full name being *Georg-August-Universität Göttingen*, founded in 1737, famous throughout Europe and the shining star of mathematics until the 1930s, and host of the best mathematicians of the nineteenth and twentieth centuries.

Göttingen, founded in 1150, is a small town of barely 120,000 inhabitants, cold and foggy in winter, but delicious in spring and summer. With narrow pedestrian streets in the center, the city is full of students and teachers (30,000 and 2,500, respectively[(3)]) which, together with the tourists and bicycles, give it a dynamic and friendly atmosphere. Its University, consisting of the historic buildings of the city center and of several new buildings, housing the 13 faculties and student services, have a great history. In winter the city is enclosed in its walls, in its libraries, in its medieval buildings. Then it is really cold and, many days, light rain soaks into the bones of lonely walkers who hear their footsteps on the ancient cobblestones.

Göttingen has the charm of the medium-sized cities in northern Europe: balanced, educated, with pedestrian streets that invite you to walk, to look at the facades and

imagine its history. In spring the city changes completely. The sun beats the mist, and a happy atmosphere installs itself. Colorful flowers decorate the house windows. The restaurant terraces take up the street. The leafy city parks become welcoming and quiet. Thousands of students flood the city with their bicycles back and forth. It's nice to see healthy youngsters, who love learning, moving around this city where everything is within reach(4).

We walk, and note the inscriptions on the plaques at the entrances of some houses: *"Carl Friedrich Gauss lived here between 1808 and 1816",* we see on one of them; *"Lejeune Dirichlet lived here between 1856 and 1859",* on another. And many more. And we are impressed to learn that the great geniuses of mathematics and physics of the nineteenth and twentieth centuries walked these streets. We stop a moment and think about where we are. We are overwhelmed by the fact that much of the mathematical history of the nineteenth century is within these walls. That here is where new hypotheses, new theorems, whole new branches of mathematics were forged, still bearing fruit to this date. That what Gauss, Dirichlet, Riemann, Hilbert, Landau and others developed here, established itself permanently. Because in mathematics, unlike other sciences, what is proved to be true, now, is true forever. It shall never be challenged. It will never go back, or be reformulated. A mathematician has the guarantee and the responsibility that his findings shall stand as the pillars of an indestructible cathedral. Once a pillar has settled down, perhaps on other pillars, it will remain forever, and the rest of the huge, endless mathematics building, will be built on this. That is the real reward of the work of a mathematician: to know that his work is eternal, that no one will ever question his new theorems, once they have been proven.

Given this responsibility, mathematicians are the most scrupulous people when making assumptions. Assumptions, which are the order of the day in other sciences, let alone in everyday life, have no room in mathematical language as an element of certainty. There are hypotheses, conjectures, which are valuable as elements of deep interest and analysis, but not as elements of truth. The truth is only achieved when that hypothesis, that conjecture is proven. If an event is verified the same way thousands, millions of times, it may be a good argument of certainty for a physicist, or for an engineer, but not for a mathematician. A mathematician needs it to be verified an infinite number of times, for all past or future times. That is why mathematicians boldly pursue to prove hypotheses and

conjectures, until confirming or refuting them. Assuming is no good, it must be proven.

Because proving, as opposed to assuming, tells us why something is so. Why it is behaving a certain way. That *why* is very important. The feeling of search sometimes becomes pathological, desperate. The truth can be hidden so deep in the foliage that we will need a machete to clear the paths. And this can take a lifetime, without ever reaching the end, but with the satisfaction of having left to other great mathematicians clear paths to move forward. Many other paths that could not be seen in that cleared path can be opened unexpectedly and cheerfully, and these could in turn lead to the discovery of more.

Mathematics bear fruit in patient societies who look at life in the long-term, who do not seek an immediate result. Which human and intellectual power, I wonder, was developed within the walls of this university? What pride must a city or a country have when appear such a fierce orchestrated eagerness to learn, when musical and literary culture, scientific knowledge, progress in mathematics, physics, chemistry, engineering, philosophy and medicine explodes in such way, leaving behind any other?

We voice all these reflections in whispers, before the facade of one of the houses where Gauss lived. We feel small before this character, whose findings and personality was infused with mathematical and astronomical history for over sixty years, and whose conjectures, too early and too valuable to be proved, were left to his successors. We give the facade of this house an admiring and respectful glance, and we exchange, knowingly, this look with other visitors who had silently joined our reflections.