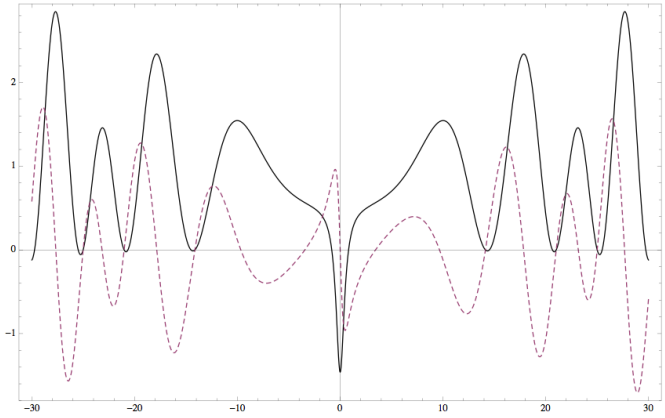

14. Forty years of assimilation

It is a well-known experience that the only truly enjoyable and profitable way of studying mathematics is the method of "filling in details" by one's own efforts.

Cornelius Lanczos (1893-1974), Hungarian mathematician.

1. The Riemann hypothesis was introduced by our hero in his famous paper of 1859. We learned of it in the last chapter, and we have now seen that proving it, after a few brief failed attempts, was not the immediate goal of the author's research. So, unknowingly, Riemann launched to the mathematical world a kind of challenge that has reached our days.

Riemann was an intuitive mathematician. He looked at the landscape before him as a generality, in its entirety. Despite the scrupulousness of his work, he was not a procedural mathematician, one of those who does not take a step in their

development until they have fully secured the one before. Riemann flew like a seagull above problems and, thanks to his global vision, he was successful, even without being totally sure if the flights he made were appropriate. The important thing is that they were right, and the proof is that, many years later, several of its unproven results were fully certified by other mathematicians.

His 1859 paper contains several obscure points[1] which are very difficult to follow. The results appear before our eyes sometimes without proof. Therefore, in the next forty years, several mathematicians took care to give greater rigor to them. It is as if this paper, full of completely new methods, would have led to later mathematicians, stroking his beard, to ask themselves: But is all this true?

Harold M. Edwards[1], on pages 4-6 of the 1974 edition of his book on the Riemann zeta function, says:

> *For the first 30 years after Riemann's paper was published, there was virtually no progress in the field. It was as if it took the mathematical world that much time to digest Riemann's ideas. Then, in a time interval of less than 10 years, Hadamard, von Mangoldt and de la Vallée Poussin succeeded in proving both Riemann's main formula for* $\pi(x)$ *and the prime number theorem, as well as a number of other related theorems. In all these proofs Riemann's ideas were crucial.*

Riemann, in his paper, had not proven the PNT, nor was it his intention. But the methods he used, which were completely new, made subsequent mathematicians wonder if it could be achieved with them. The methods used by Chebyshev were not the right ones, as Godfrey H. Hardy would say already in the twentieth century. But Riemann's methods, based on the analysis of functions of complex variable, allowed to see things from another very different point of view.

2. When Riemann presented his 1859 paper, Schopenhauer's philosophy had been discovered by Richard Wagner, which led him to compose his work *Tristan and Isolde*: an erotic scandal. Wagner was a success in the opera houses,

along with Verdi, who had already premiered *Nabucco* and *Il Trovatore*. Meanwhile, the virtuoso genius Franz Liszt had just composed his *Symphonic Poems*, and seven years later Johann Strauss, the son, composed his beautiful *Blue Danube*. France read *Madame Bovary*, a realistic novel written by Gustave Flaubert two years earlier, and Europe danced the waltz in their ballrooms, even in times of war, while preparing for the unification of Germany in 1871.

In 1866 the Third War of Independence laid the foundation for the unification of Italy in 1867, and it was not precisely to the rhythm of a waltz that Karl Marx published *Capital* in London. This philosophical, economic and political work would stir societies until the end of the twentieth century. That same year, across the Atlantic, Thomas A. Edison began his scientific career, which eventually illuminated everyone. Jules Verne began writing in those years his series of novels on extraordinary trips, and took us to the center of the Earth, to the Moon and to the bottom of the sea, while Victor Hugo had written *Les Misérables*.

Paul Cézanne began his Impressionist period in 1870, alongside Pierre-Auguste Renoir. The Montmartre in Paris became the capital of the Bohemians, and a changing world, more tolerant, began to grow in a humiliated France after being defeated by Germany. In the United States John D. Rockefeller was already gathering an impressive fortune thanks to oil, while Andrew Carnegie had become the master of steel. And in Germany, in 1879, Albert Einstein was born.

The proclamation of the German Empire in the Hall of Mirrors of the Palace of Versailles, January 18, 1871. King Wilhelm I of Prussia is named Kaiser of the German Empire, and is honored by the princes who represent the new states of the Empire.

Painting of 1877 by Anton von Werner.

In times of Tsar Alexander III, in the 1880s, Russia was moved listening to the music of Tchaikovsky, whose *1812 Overture* reaffirmed patriotic feelings, while chemistry students learned the *Periodic Table*, established ten years earlier by Dmitri Mendeleev. Science began to apply methods in a way never seen before. Medicine and biology, through the microscope, had a huge breakthrough, and Louis Pasteur saved for the first time the life of a child infected with rabies in 1885. Vaccinations began to develop, and with them life expectancy began to lengthen in the West. In March 1889, it was the centenary of the French Revolution, the Eiffel Tower was inaugurated in Paris.

In the 1890s *Verism* succeeded in Italy, and Gustav Mahler finished his *Titan symphony* in Leipzig (another scandal, but now artistic, since it *defied all laws of music*). The *Dreyfus Affair* moved France, Vincent van Gogh had recently passed away, and Ramón y Cajal conducted research on neurology in Barcelona, which led him to winning the Nobel Prize for Medicine in 1906. Sigmund Freud invented psychoanalysis in Vienna, which changed the way we understand psychiatry. Richard Dedekind, Riemann's friend and old acquaintance of ours, retired in 1894 in Braunschweig. And Spain lost Cuba in 1898 in an awkward and unequal war against the United States. The Nineteenth century was coming to an end, plagued by wars, revolutions, discoveries, much science, economics, social advances and new freedoms. And another century began, the twentieth century, even more complex.