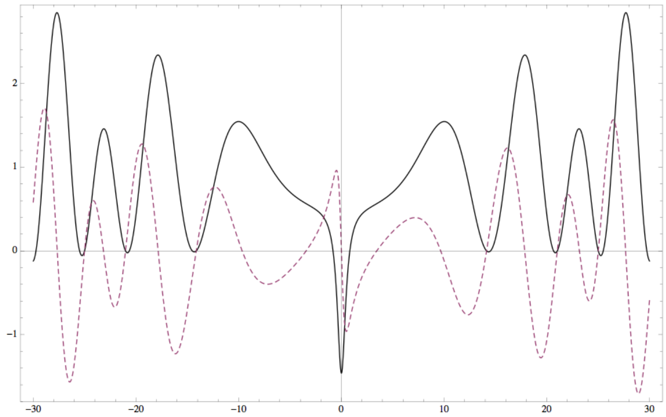


23. The Experts Speaking

Wer ein mathematisches Buch nicht mit Andacht ergreift, und es wie Gottes-Wort liest, der versteht es nicht.

He who takes a math book without devotion, and he who does not read it as if it were the word of God, will not understand It.

Novalis (1772-1801), German poet.

1. A trip to Minnesota and another one to California. I will end this book as I began it. Traveling. Because the history of the Riemann hypothesis is a long and beautiful trip throughout more than three centuries, that has not come to an end.

From Barcelona I have prepared meetings with two of the most renowned mathematicians today, specialists in the Riemann *zeta* function and hypothesis:

one, with Andrew M. Odlyzko, a professor at the University of Minnesota; and another with J. Brian Conrey, director of the *American Institute of Mathematics (AIM)*. The two have worked with enthusiasm for many years in the knowledge of the Riemann *zeta* function. Thanks to them, and through their many published articles, which have become legendary, we now know this rebellious function better than ever.

Both mathematicians have given me ideas and ways of exposure that have been useful to me when I revised some of the chapters of this book. My meetings with them will be useful to round it up. My friend Joan Marcos, a mathematics lover, accompanied me during my travels, as could not be otherwise, and was present at the meetings in a very productive way. I must confess that, before these meetings, I was nervous, because Odlyzko and Conrey are two giants of mathematics, and I am a mere chronicler. But they were very kind, and made it very easy for me, with an enthusiasm that I am grateful for.

Andrew Michael Odlyzko

2. Andrew Michael Odlyzko (Tarnów-Poland, 1949). We had scheduled a meeting with him at 12.00 on a Monday in his office in the *Vincent Hall,* the School of Mathematics building at the University of Minnesota. Before this we had taken a good walk around the campus, and were surprised by the university's modernity and vitality. The equipment and infrastructure are formidable. This university has produced nine Nobel laureates, two Pulitzer Prizes, two vice presidents of the United States, 56 senators, many scientists, liberal professionals, businessmen, doctors, and, although he did not graduate, Bob Dylan also studied here. We note a lively student atmosphere, with young people here and there, all dedicated to one thing: to train themselves. Here we have not seen anyone wasting time.

We meet Odlyzko. First, we eat lunch at one of the shared services building, and then we go back to his office. He is a well-known mathematician, not only in academia, but also for his work in data encryption, economic studies, articles on

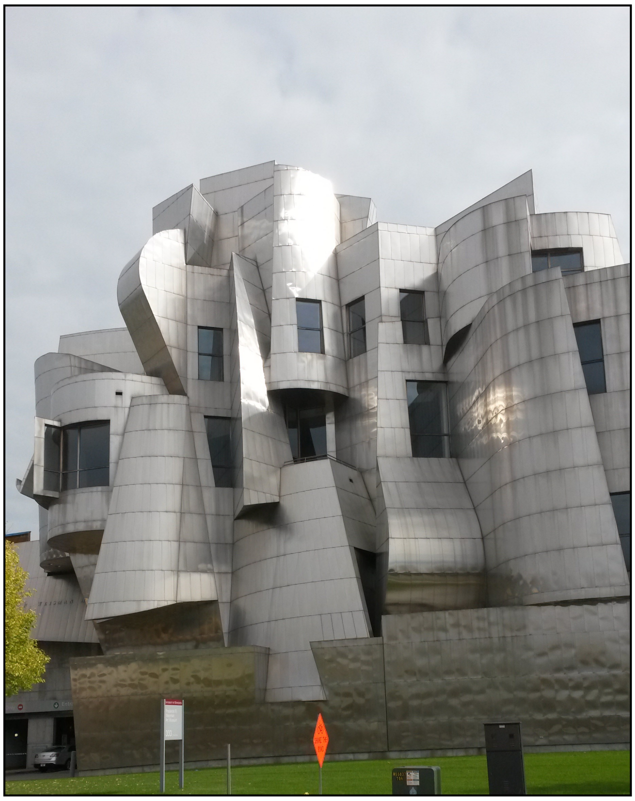
The Vincent Hall building, headquarters of the School of Mathematics at the University of Minnesota.

Photo by the author.

theory of communications and on Internet traffic. He is often interviewed by the press, such as the *Wall Street Journal* and the *Forbes* magazine, where he has outlined his knowledge of various subjects, such as the growth of data traffic on mobile networks, or the inefficiency of the British railway network in the nineteenth century.

2. His studies, Bell Labs and the Mertens conjecture. Of Polish origin, in 1963 Odlyzko came to the United States with his parents, at the age of 14, where he finished high school. He carried out his college period at the *California Institute of Technology (CalTech),* and graduated with a PhD in mathematics from the Massachusetts Institute of Technology (MIT). He immediately got a contract at *Bell Labs*, then a unit of AT&T, and now part of Nokia (and before that, Alcatel-Lucent), where he spent 26 years.

Bell Labs is a company with a fascinating history. It is not only engineering, but pure research, where physicists and mathematicians have made real advances, such as the invention of the transistor in 1947, the solar cell in 1954, the Laser in 1957, the first communications satellite in 1962 or the development of the UNIX operating system and the C programming language in 1972. Bell Labs has produced eleven Nobel laureates (8 awards in total), which gives an idea of the scientific level of the company, which has allocated huge amounts of money for research, often sunk.



Detail of the Weisman Art Museum, University of Minnesota, designed by Frank Gehry.

Photo by the author.

Odlyzko worked at Bell Labs in mathematical research, coding theory, communication systems and data encryption, and held relevant executive positions. He is not considered a pure mathematician, but an applied one, although his work on the *zeta* function may have associated him with pure theory. He devotes practically all the year to research. He likes to publish articles in collaboration with other mathematicians, and he has over a hundred. When asked what his Erdös number was, he immediately answered: it is 1.

We address the issue of the Mertens conjecture, and I ask him about when he started to think about its falsity. Andrew tells me that very soon, in 1971, before graduating. Already when he was at Bell Labs, in 1983, he decided to prove it. He took as base the previous work of Albert Ingham (1900-1967), in which the latter seriously doubted the certainty of the conjecture. The proof is difficult to outline, given its deep mathematical level. It consists of a part of analytic theory, and of another part of numerical calculation using the zeros of the *zeta* function.

Odlyzko was interested in the fine calculation of zeros, because, for the proof, he had to calculate the first 2,000 zeros, with a precision of 100 decimals. When he published an article in 1985, together with the Dutchman Herman J. te Riele (The Hague, 1947), it was known that the first 300 million zeros were on the critical line. But one thing is knowing that a zero is on the line, for which it takes very little accuracy, and another thing is knowing a large number of decimal places (let us recall that to know if a zero is on the line, it is enough to know whether the $\xi(1/2 + it)$ function, that is already known, and that is real when t is real, crosses the line).